

A MODEL FOR THE CHARACTERIZATION OF DAMAGE IN CYCLICALLY LOADED STONES

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SOMMARIO

Il comportamento dei materiali lapidei sottoposti a cicli di compressione monoassiale viene interpretato alla luce di un modello semplice, basato su considerazioni di meccanica statistica. Partendo dall'osservazione sperimentale che il danneggiamento provoca scorrimenti a taglio lungo piani inclinati rispetto alla direzione di carico, si immagina che tali scorrimenti dipendano dal valore medio della tensione tangenziale. Su questa influiscono le concentrazioni di sforzo inevitabilmente presenti all'interno del materiale le quali, essendo dovute ad eventi casuali quali micro-fessurazioni, micro-inclusioni etc., si ipotizza che possano essere rappresentabili statisticamente. Stabilendo così un bilancio fra la probabilità che lo sforzo tagliente raggiunga il valore limite in un determinato piano e la distribuzione statistica delle resistenze dei vari possibili piani di scorrimento, si ricava che la contrazione assiale del provino è una funzione lineare del logaritmo del numero dei cicli. Questo risultato è confermato dall'evidenza sperimentale.

ABSTRACT

The behavior of natural stones subjected to cyclic uniaxial-compression loading is interpreted through a simple model based on statistical mechanical considerations. We take as a starting point the experimental observation that localized damage produces the slipping of planes inclined with respect to the loading direction and presume that such slippage depends on the average value of the shear stress. This value is strongly affected by the stress concentrations inevitably present within the material which, being due to random events such as micro-cracks, micro-inhomogeneities etc., can be represented statistically. Thus, a balance is established between the chance that the shear stress in any given layer reach the limit value, and the statistical distribution of the strengths of all possible slip layers. In this way, we find that axial contraction in the loading direction results to be a linear function of the logarithm of the number of cycles. The experimental evidence confirms this trend.

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1. INTRODUCTION

The mechanical response of natural stones when subjected to cyclic uniaxial compression has recently been considered in a wide-ranging experimental study [6]. Cylindrical specimens were tested through varying sequences of two-level programmed loading, during which strains were measured by gauge-rosettes placed on the specimens' lateral surfaces. A typical diagram for a cyclic test on Carrara marble is presented in Figure 1, where the average stress σ is plotted as a function of two different strain components, ϵ_v and ϵ_h , measured respectively in the vertical and horizontal directions, that is, parallel to the loading direction or orthogonal to it. As can be noticed, a typical feature of the fatigue response is that repetition of the loading cycle produces a progressive accumulation of permanent strain, rather than any significant decay in either the elastic modulus or Poisson's ratio. Such behavior is common not only to natural conglomerates, but also to artificial ones as well (concrete), though in the latter case, the inelastic deformation is usually accompanied by decay of the elastic modulus [1].

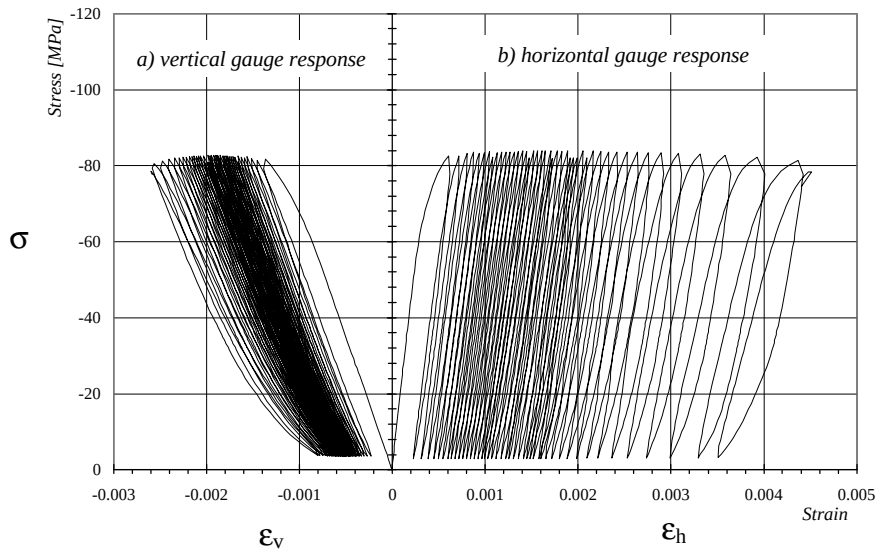


Figure 1. Typical σ - ϵ relations for cyclic-compression tests on Carrara Marble.

Further processing of experimental data led to the inference, albeit a tentative one, that the inelastic portion of strain in the loading direction, ϵ_v , can be considered a natural macroscopic indicator of the evolution of damage. Much evidence supports this conjecture. In [6], a strict correlation was revealed between the energy dissipated in each hysteresis loop and the increase in the permanent increment of ϵ_v in the same cycle. Secondly, the ultimate contraction in the loading direction appears to be the single parameter which best characterizes the material's performance: regardless of the load history experienced, failure occurs when this parameter reaches a certain limit, characteristic of each rock type (usually termed “strain capacity”). What is more, by maintaining the loading intervals at each cycle fixed, the permanent deformations exhibit what is to be considered a “steady trend”. This assertion stems from one observation. If values of ϵ_v are measured in correspondence to the minimum (in absolute value) load at each loop, and these values are plotted as a function of the relative cycle number n , we obtain graphs like that in Figure 2 (on a semi-logarithmic scale). Since the minimum load level is kept sufficiently small, such strain measurements are representative of the permanent deformation accumulated during the ongoing cycles. Thus,

such graphs evidence that, for most of the damage evolution, inelastic strain shows a pseudo-linear dependence on the log of n , at least until the final stage immediately preceding rupture is reached. The slope of this interpolation line is a significant parameter, strictly correlated with the material's underlying microstructure [3].

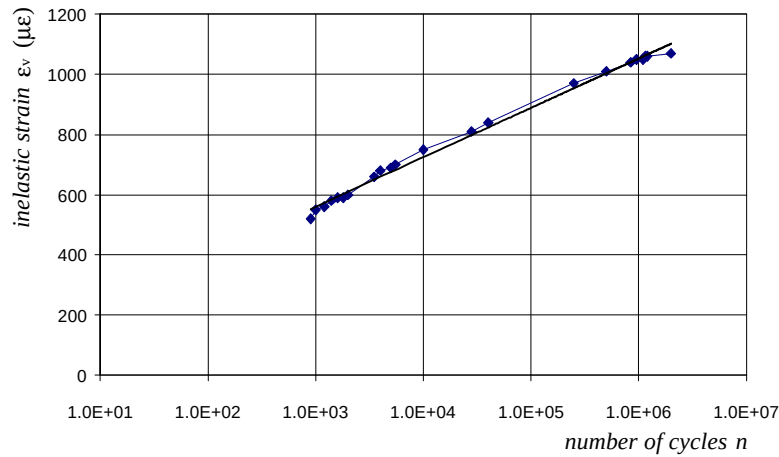


Figure 2. Linear relationship between inelastic strain and $\text{Log}(n)$.

Processing of the output of the strain-gauge rosettes has moreover revealed that accumulation of permanent deformation is essentially due to a shear-like mechanism. Despite the brittleness traditionally attributed to rocks, slip planes inclined approximately 45° to the horizontal develop throughout the specimen. Such slip planes were evidenced by Prandtl and Rinne ([5], pp. 330, Figure 10.4) by pencil etchings of the polished lateral surface of marble loaded under uniaxial compression. Further studies using a scanning electron microscope [2] have provided evidence of the correlation of inelastic deformation to a generalized “loosening” of the rock, produced by apparent “decohesion” of its constituent granules. In particular, the constituent calcite grains remain integral, but detach one from the other without breaking along cleavage planes, as if the cementing material between them were being gradually destroying (Figure 3a). In passing, it is worth noting that if the same material is broken with a chisel, the resulting fracture surfaces (Figure 3b) display considerably varying shapes, characterized by grains broken along cleavage planes with hardly any sign of decohesion.

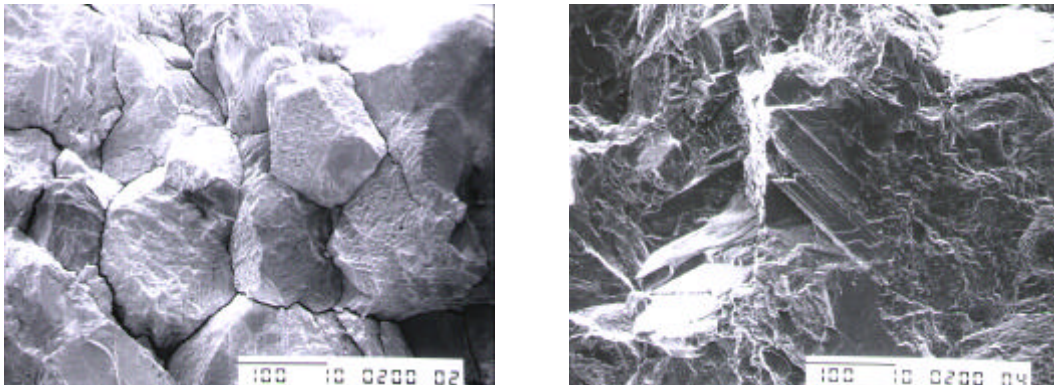


Figure 3: Fracture surfaces of Carrara marble: a) cyclically broken, b) broken with a chisel.

The aim of this note is to develop a model able to interpret the fatigue behavior of natural stones, in particular the pseudo-linear dependence of permanent strain on the logarithm of the number of cycles n , as shown in Figure 2.

2. THE MODEL

One of the principal hypotheses consists in assuming that the increment in inelastic deformation in the loading direction is produced by a series of events that are *independent* one from the other. In other words, the occurrence of a process in a certain particle does not influence the behavior of the neighboring constituent elements. Two arguments uphold such a hypothesis. At the microscopic level, SEM observations [2] have shown that the degradation mechanism is essentially due to decohesion of the constituent grains (Figure 3a). Hardly any transgranular cracking at all can be found, so the process can be considered to occur at a length scale of the same order of magnitude as the diameter of a unit grain. Further evidence comes from observations, already mentioned in the Introduction, that permanent deformation is essentially produced by the slipping of shear layers. Such slippage does not occur at the atomic level, as in the theory of metal-lattice dislocations, but is rather produced by simultaneous infinitesimal rotations of the grains laying on the same layer, which can only occur when the bonding between grains, as in Figure 3a, has been sufficiently loosened. Figure 4, taken from [5], shows a possible system of slip layers in a compressed specimen. By this view, planes can slip practically *independently* of each other. Permanent contraction in the direction of loading is the result of the consecutive activation of different layers.

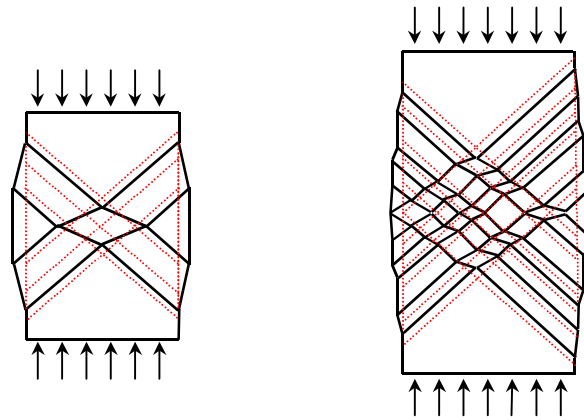


Figure 4. Possible shear layers in compressed bodies.

Therefore, by our way of thinking, the basic process producing inelastic deformation consists in activation of a single slip plane. In particular, we may suppose that slippage occurs when the average shear stress in the considered plane, say t , equals a certain activation stress t_a . We further assume that slippage of one plane produces a relative displacement of the specimen's ends by a certain quantity v_p so that, if h denotes the height of the sample,

$$e_p = \frac{v_p}{h} \quad (1)$$

represents the plastic contribution to the specimen's overall axial strain due to the slipping of *one* plane. We may furthermore suppose that each plane can yield only once, that is, its activation stress becomes very large (practically infinite) after it has flowed. In fact, after the texture has been loosened the grains can rotate infinitesimally, but once the grains have settled down in a new stable position, it takes much higher stresses to make them move further.

Now, taking into account all these assumptions, we want to characterize the material response when the specimen is cyclically loaded. For the sake of convenience, stress and strain will be considered positive when corresponding to compression, and we will examine the case of mean axial stress varying between the limits $\sigma_d \geq 0$ and $\sigma_u \geq \sigma_d$.

The key-point in the analysis is recognizing that in real materials, especially rocks, various circumstances, such as micro-inhomogeneities, micro-cracks, micro-inclusions or thermal variations, produce a system of self-balanced micro-stresses in addition to the applied bulk stress. In order to account for such effects relative to the possible slippage of one layer, we assume that a class of functions

$$D^{[s]} = D^{[s]}(t) \quad (2)$$

is defined, representing the number of planes where the average shear stress is greater than t when the mean compressive stress in the sample equals σ . Moreover, denoting by N_{tot} the total number of layers in the specimen where slippage may possibly occur, we introduce the probability function

$$P^{[s]} = P^{[s]}(t) = \frac{D^{[s]}(t)}{N_{tot}}, \quad (3)$$

which, being equal to the ratio between the number of favorable events and the total number of events, also represents the *probability* that a certain *fixed* plane undergoes shear stress greater than τ when the applied mean stress equals σ .

The yielding of a certain number of layers when local stresses reach the activation value can produce a significant re-distribution of microstress within the specimen. However, for the reasons stated above, we assume that whatever the load cycle, the number of elements whose stress exceeds t is still given by an expression similar to (2). In other words, (2) and, consequently (3), are considered not to vary over the ongoing cycles and are thus the same at any cycle number, even if the *distribution of microstress* may vary randomly between one load cycle and the next. Moreover, it is reasonable to assume that the most severe stresses in each cycle are reached at the peak average stress σ_u , so that the probability function (3) has σ_u as an apex when considered within each cycle. On the other hand, because of the independence of the considered events, the probability that a certain fixed plane will undergo a stress greater than τ in n cycles is the sum of the probabilities that such an event will occur in any one of the cycles, that is, recalling (3),

$$nP^{[s_u]}(t) = \frac{nD^{[s_u]}(t)}{N_{tot}}. \quad (4)$$

In particular, by assuming that the number of cycles performed is so high that n can be considered to be a continuous variable, the quantity

$$\frac{\partial}{\partial n} [n P^{[s_u]}(t)] dn = P^{[s_u]}(t) dn, \quad (5)$$

represents the chance that a certain given layer will experience a shear stress greater than τ during a cycle-number increment dn .

Now, let

$$\Pi(t', n) dt', \quad (6)$$

stand for the number of planes whose activation stress τ , at cycle number n , ranges between the limits $\tau' < \tau < \tau' + d\tau'$. The reduction of this quantity over the cycle increment dn will result to be equal to the number of elements yielding at the same time. Thus, considering this balance in statistical terms we obtain

$$-\frac{\partial}{\partial n} [\Pi(t', n)] dn dt' = \Pi(t', n) P^{[s_u]}(t') dn, \quad (7)$$

that is,

$$\frac{\partial}{\partial n} [\Pi(t', n)] = -\Pi(t', n) P^{[s_u]}(t'). \quad (8)$$

This equation can be integrated to give

$$\Pi(t', n) = \Pi(t', 0) \exp[-P^{[s_u]}(t') n]. \quad (9)$$

Having assumed in (1) that the yielding of any one plane produces an increment ϵ_p in the overall strain, we consequently find, from (8) and (9), that the increase in mean axial strain per unit cycle takes the simple form

$$\frac{de}{dn} = e_p \int_{all t} -\frac{\partial}{\partial n} [\Pi(t', n)] dt' = e_p \int_{t_{min}}^{+\infty} \Pi(t', 0) P^{[s_u]}(t') \exp[-P^{[s_u]}(t') n] dt'. \quad (10)$$

Here, t_{min} represents the minimum value of the activation stress amongst all the possible values; the maximum activation stress, on the other hand, has been assumed to be infinite.

We can expect, at least as a first approximation, a Gibbsian canonical probability distribution for (3), which takes the form

$$P^{[s_u]}(t') = \exp\left[\frac{\Psi - g^{[s_u]} t'}{\Theta}\right]. \quad (11)$$

In this expression, according to [4], chapter IV, the constant Θ denotes the distribution modulus, whereas $g^{[s_u]}$ is a material parameter, now representative of the micro-stress distribution inside the specimen. The apex is a reminder that this parameter may possibly depend on the maximum applied stress σ_u . The other constant Ψ can be found by a condition of the type

$$P^{[s_u]}(t') \Big|_{t'=t_{\min}} = 1 \Rightarrow \Psi = t_{\min} g^{[s_u]}, \quad (12)$$

which expresses the fact that when the stress σ_u is applied, all the layers experience a shear stress greater than or equal to $\tau_{\min}$. In other words, the applied stress σ_u is high enough that some layer will certainly yield during the first load cycle.

Noticing that

$$dP^{[s_u]} = -\frac{g^{[s_u]}}{\Theta} P^{[s_u]}(t') dt', \quad (13)$$

we can, by changing variables in (10) and recalling (11) and (12), obtain

$$\frac{de}{dn} = e_p \frac{\Theta}{g^{[s_u]}} \int_0^1 \Pi(t', 0) \exp[-P^{[s_u]}(t') n] dP^{[s_u]}. \quad (14)$$

In the simplest case, we may assume that the activation stress is uniformly distributed in virgin material ($n=0$), so that

$$\Pi(t', 0) = \text{const.} = \Pi_0. \quad (15)$$

Thus, equation (14) can be integrated to give

$$\frac{de}{dn} = e_p \frac{\Theta \Pi_0}{g^{[s_u]}} \frac{1 - \exp(-n)}{n}. \quad (16)$$

But for sufficiently large n (actually, greater than just a few units), we have, in general, $\exp(-n) \ll 1$. Consequently, from (16), with very good approximation,

$$\frac{de}{dn} \cong e_p \frac{\Theta \Pi_0}{g^{[s_u]}} \frac{1}{n}. \quad (17)$$

Integration of this last equation yields

$$e(n) = e_p \frac{\Theta \Pi_0}{g^{[s_u]}} \ln(n) + A, \quad (18)$$

which is the anticipated logarithmic dependence.

3. DISCUSSION AND CONCLUSIONS

Formula (18) expresses the material's behavior when subjected to cyclic loading. The quantities ε_p and Π_0 appearing therein are clearly material parameters, unlinked to the testing conditions. On the other hand, constant A is representative of the strain increase consequent to the initial settling of the specimen, occurring as a rule in the very first cycle. This parameter may depend upon the most disparate causes (i.e. bedding errors, imperfect planarity of the specimen's bases, etc.) and should consequently be considered a random variable. The results of the tests performed do in fact exhibit wide scattering as far as this parameter is concerned, confirming a non-continuous dependence on the macroscopic testing conditions [3].

The parameters Θ and $g^{[s_u]}$, instead, have a profound physical significance. Recalling the expression and significance of $P^{[s_u]}(t')$ represented in (11), the greater this quantity is, the higher is the probability that a certain layer will be subjected to a shear stress greater than t' when the bulk axial stress is σ_u . In other words, the greater this number is, the greater must be the "amount of defects" (micro-cracks, micro-inclusions, etc.) present in the material..

Consequently, increasingly heterogeneous materials are characterized by increasingly large values of the function $P^{[s_u]}(t')$, corresponding to great Θ s and small g s. Moreover, it is clear that

$$s_1 < s_2 \Rightarrow g^{[s_1]} > g^{[s_2]} . \quad (19)$$

In fact, the higher the applied normal stress is, the greater the probability that layers be subjected to high shear stresses.

Naturally, the foregoing represents only a first attempt at theoretically modeling a very complex phenomenon. Further developments could consider, in particular, more complex expressions for parameter $g^{[s_u]}$, ones that depend, not only on the maximum, but on the minimum value of the stress interval as well. In addition, the static ultimate strength of the material may also enter into the other parameters, Ψ and Θ , appearing in (11).

As a final remark, it is worth stressing once again that the central assumption underlying all the above considerations is that of the *statistical independence* of the events causing the effect as observed at the macroscopic level.

REFERENCES

- [1] Comité Euro-International du Béton, *Fatigue of Concrete Structures: State of the Art Report*, Bulletin N. 188, CEB, 1988.
- [2] DalPino, R., Narducci, P., Royer-Carfagni, G., A SEM investigation on the fatigue damage of marble, *Journal of Material Science Letters*, in press, 1999.
- [3] Fantuzzi, C., Royer-Carfagni, G., On the characterization of the fatigue response of natural Stones by Dimensional Analysis, work in progress, 1999.
- [4] Gibbs, J. W., *Elementary Principles in Statistical Mechanics*, Dover, New York, 1960.
- [5] Nadai, A., *Theory of Flow and Fracture of Solids*, Vol. I, McGraw-Hill, New York, 1950.
- [6] Royer-Carfagni, G., Salvatore, W., The characterization of marble by cyclic compression loading: experimental results, *Studi e Ricerche*, Dipartimento di Ingegneria Strutturale dell'Università di Pisa, rapporto n. 4, 1998.