PARTIAL SAFETY FACTORS IN DIMENSION STONE DESIGN

Rui S. Camposinhos[1], José C. Lello[2]
[1] *Structural Engineer – PhD, School of Engineering, Polytechnic of Porto, Portugal, rdc@isep.ipp.pt*
[2] *Structural Engineer – MSc, School of Engineering, Polytechnic of Porto, Portugal, jca@isep.ipp.pt*

Abstract
Today's thin stone veneers must be designed to withstand, besides the self weight, high wind pressures and seismic forces. They must accommodate hygrometric expansion and contraction or other differential movement, deflection, vibration and creep, etc. When available, code requirements do not provide an accurate or realistic safety factor for the specific type of anchor and or stone used in a project. In fact safety factors for stone had always been conservative compared with actual safety factors for "uniform" materials like steel or concrete. Traditionally, safety factors have been applied by comparing the ratio of the "design" strength to the "design" stress (load) with standard or accepted values for the stone and or situation which is being designed. Their values are subjective taking into account the rupture stresses of stone itself. Safety global factors recommended by stone industry associations have a wide value range and in comparison with the equivalent values in modern codes are much lower. Over the past five decades, there has been a gradual move towards limit state or load/resistance factor design. This approach requires the fulfilment of two principal criteria: the ultimate limit state (ULS) and the serviceability limit state (SLS). Assuming suitable classes of risk according Eurocode EN 1990 for the basis of structural design and by means of appropriate expressions partial factors of safety are proposed in accordance with limit state principles for dimension stone design.
Keywords: dimension stone, safety factors, reliability analysis.

1. Introduction

Dimension stone has its own unique qualities that differentiate it from "man-made" materials. It is not manufactured; blocks are removed from the quarry, slabs are cut from these blocks, and the slabs are further sawn into the final stone to be mounted after eventual face finishing. Each block is different; each slab is different. In contrast to the uniformity of materials produced by machine or assembly line, dimension stone's naturally varied appearance is a magnificent work of art. The term "uniformity of material," when applied to natural stone has relative meaning.

Today's thin stone veneers must be designed to accommodate hygrometric differential movement, deflection, vibration and creep, etc. And to resist, besides the self weight, high wind pressures and seismic forces. When available, code requirements do not provide an accurate or realistic safety factor for the specific type of anchor and or stone used in a project. In fact safety factors for stone had always and still are used as rules of thumb compared with actual safety factors for "uniform" materials like steel or concrete. Testing for mechanic and physic characterization in "man-made" materials with controlled production such as steel or concrete are common procedure, yet, with natural stone where the scatter of the material's properties is much more evident this procedure is limited to only some tests in type or in quantity. Furthermore, other "nature-made" materials such as timber are much better placed in this context than natural stone products.

2. State of Art

The design method for dimension stone based on allowable stresses has been and, it can be said, still is the most generally used in dimension stone. This method, itself, contains no rules for defining these allowable stresses.

Traditionally, safety factors have been applied by comparing the ratio of the "design" strength to the "design" stress (load) with standard or accepted values for the stone and or situation which is being designed. Their values are subjective taking into account the rupture stresses of stone itself.

Safety global factors recommended by stone industry associations range from 3 to 8 [1; 2; 3]. When comparing the equivalent values in modern codes we can find that the so called partial factors are much inferior, yet the level of safety sometimes does not come along with it [4].

One reason stone safety factors are more conservative is that stone is a natural material and not closely controlled, manufactured product. The physical properties of stone, even for the same type of stone and quarry, can vary

widely. The attempts to improve design methods bases on allowable stresses in order to obtain a better definition of load and resistances did not succeed showing in practice that all the data and results are disperse [4]. It must be said that these global safety factors had been incorrect and persistently misunderstood; these factors are sometimes derived from mean values of material's strength, disregarding their dispersion against some probabilistic value of loads or actions.

It must be emphasized that dispersion of material's properties is rather important even for elements considered similar. The need to use statistical approach for studying the problem of structural safety in dimension stone design is obvious.

Over the past five decades, there has been a gradual move towards limit state or load/resistance factor design for the so called construction "man made" materials. This approach applies factors to each component of the design assumptions to accommodate unknowns. Limit state design requires the "structure" to satisfy two principal criteria: the ultimate limit state (ULS) and the serviceability limit state (SLS). A limit state is a set of performance criteria (e.g. vibration levels, deflection, strength, stability, buckling, twisting, collapse) that must be met when the structure is subject to loads or any type of direct or indirect action.

Limit state design has replaced the older concept of permissible stress design in most forms of civil engineering. As a result, all modern buildings are designed in accordance with a code which is based on limit state theory.

In 1975, the Commission of the European Communities (CEC), decided on an action programme in the field of construction regarding the elimination of technical obstacles to trade and the harmonization of technical specifications and took the initiative to establish a set of harmonized technical rules for the structural design of construction works: the Eurocodes. There are 10 structural Eurocodes covering the design of structures of concrete (EC2), steel, (EC3), composite steel and concrete (EC4), timber (EC5), masonry (EC6), aluminium (EC9). The other Eurocodes cover the basis of structural design (EN 1990) actions (EC1), the geotechnical design (EC7), and the design of structures for earthquake resistance (EC8). There is an astonishing absence of "the stone" material in these programme and at almost if not all the countries existing rules may be considered archaic or at least obsolete.

3. Risk and Reliability Analysis

In the assessment of civil engineering structures the capability of a designed system, to respond to project requirements or to meet user's demands, must be assessed but without not taking care of some basic but fundamental principles: a defined level of risk has to be accepted subjected to economic and social constraints. To establish the point at which a system ceases to operate is to a certain degree subjective – this decision involves itself uncertainty and the decision makers must balance the benefits against costs.

In fact a system can fail to perform its intended function for one or more reasons, such as natural hazards or lower performance than predicted. Failures may even include such rare events as collapse of major structures. Although the assurance of a system safety is primarily a task of engineers, the accepted levels of risk are subject to economic and social constraints and in this way social issues play an important role in the analysis of civil engineering systems, because these systems are more directly involved with the public than other engineering systems.

Determining the stage at which a system ceases to perform adequately is to a certain degree subjective. The inherent decisions involve some additional uncertainty in the definition of system performance and safety, as well as the judgment of decision makers, who must balance the benefits against costs.

To analyze a system's risk of failure, one must clearly identify the input to the system and its consequent response. In the case of a building, structural safety depends on the maximum load that may be imposed over the lifetime of the building, and also on the load-carrying capacity, or strength, of the structure or its components. This is a general problem and must be applied to cladding too. In fact the predictions of maximum load and actual strength of stone cladding is subject to the same uncertainties; one cannot ensure its absolute safety, and the engineer had to rely on some probabilistic concept indicating the likelihood that the available strength will adequately withstand the maximum load over the lifetime of the facade cladding system.

By this reliability is defined as the probabilistic assessment of the likelihood that a given system will perform adequately for a specified period of time under known operating conditions, or more

simply reliability, *r*, of a system is defined as the probability of nonfailure over a specified system lifetime. In the opposite sense one has the definition of risk as the probability of failure in the same conditions.

In this way the risk that a system is incapable of meeting the demand is defined as the probability of failure, over the specified system lifetime under specified operating conditions. System reliability, denoted by in Equation 1, is the (complementary) probability of nonfailure.

$$r = 1 - P_f \qquad (1)$$

where P_f is the probability of failure.

4. Measures of Reliability

As already mentioned the assessment of risk and safety in cladding design are traditionally based on allowable factors of safety; these are estimated from previous experience with the behaviour of a particular fixing system or from observed behaviour of similar systems. An usual measure of the safety factor taken by the designers is the ratio of the assumed nominal values of the material strength, say resistance, *R** and action, S*

$$F^* = \frac{R^*}{S^*} \qquad (2)$$

For example, if the allowable stress in a dimension stone slab is 4.0 MPa and the design stress due to wind load is 3.0 MPa, the conventional safety factor is 1.33. The engineer may assume that the designed thickness slab is satisfactory if the calculated safety factor is greater than an accepted minimum value, which can be based on experience or by design imposed prescriptions. If a safety factor of 1.33 is regarded as low, the engineer should redesign the slab by two different procedures: increasing the thickness and thus its capacity; decreasing the support distance, and consequently reducing induced wind stress.

Because the nominal values of both resistance, and action cannot be assessed with certainty, the capacity and demand functions, *R*, S** has to be considered as probability functions. Hence, the safety factor, given by the ratio *R/S* of two random variables {*R;S*} is also a random variable.

The inadequacy of the system to meet a required demand, measured by the probability of failure, is associated with that portion of the distribution of the safety ratio wherein it becomes less than unity, that is a given structural element will be considered to have failed if its resistance is less than the stress resultant. The probability of failure of the structural element can be sated as follows:

$$P_f = P(R \le S) = P(R - S \le 0) \qquad (3)$$

Defining $f_S(s)$ and $f_R(r)$ as the probability density independent functions of demand, S, and capacity, R, the probability, P_f, of system failure is given by:

$$P_f = P(R \le S) = \int_{-\infty}^{\infty} \int_{-\infty}^{s=r} f_R(r) f_S(s)\, dr \times ds \qquad (4)$$

For common, but special case when S and R are independent equation (4) can be written in the single integral form, as is has no physical meaning to define the distributions to negative values of a variable x:

$$P_f = P(R \le S) = \int_{0}^{\infty} F_R(x) f_S(x)\, dx \qquad (5)$$

For only some distributions of R and S it is possible to integrate the convolution integral in (5) analytically. An example is when both are normal random variables with means m_R and m_S and correspondingly variances s_R^2 and s_S^2. The safety margin given by $Z = R - S$ has a mean and variance given by well-known rules for addition (subtraction) of normal random variables:

$$m_Z = m_R - m_S \qquad (6)$$

$$s_Z^2 = s_R^2 + s_S^2 \qquad (7)$$

Substituting (6) and (7) in Equation (3) one has:

$$P_f = P(R - S \le 0) = P(Z \le 0) = \Phi\left(\frac{0 - m_z}{s_z}\right) \qquad (8)$$

$\Phi(\,)$ is the very well-known standard normal distribution from statistics. The random variable $Z = R - S$ is shown on Figure 1, in which the failure region $Z \le 0$.

It must be regarded that the probability of failure is the probability of $R - S$ taking negative values, hence using equation (6), (7) and (8), it follows that this probability is equal to the probability of obtaining in a normal distribution, values less than the mean at a distance higher than $\frac{(m_R - m_S)}{\sqrt{s_R^2 + s_S^2}}$, measured in standard deviations.

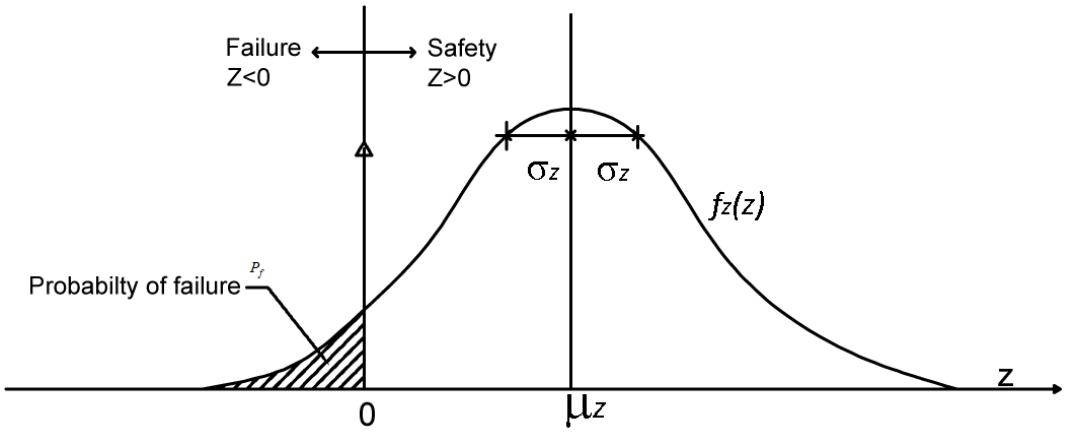

Figure 1 – Distribution of safety margin Z=R-S.

5. Reliability Index

Consider Equation 8 rewritten in the form:

$$P_f = \Phi\left(-\frac{m_z}{s_z}\right) = \Phi(-\beta) \qquad (9)$$

where β is the so called reliability index, one of the most frequently used reliability measures of structures, e.g., EC 1990. It should be underlined that this reliability index is completely equivalent to the probability of failure whatever method is used for its calculation, for which it may be explicitly expressed:

$$\beta = -\Phi^{-1}(P_f) \qquad (10)$$

where $\Phi^{-1}(P_f)$ denotes the inverse distribution function of the standardized normal distribution of the probability of failure. Note that the negative sign on the right-hand side of (10) is introduced to keep β positive for values of the P_f, less than 0.5.

It follows from (8) and (10) that in the considered case, based on the assumption of a normal distribution for both, the load effect, S, and the resistance, R, the reliability index is given by:

$$\beta = \frac{m_z}{s_z} \qquad (11)$$

Let us consider again the dimension stone slab from the previous example in which the lateral action induced stress at mid-span has a mean value $m_w = 3.0$ MPa and for example a variance $s_w^2 = 1.0$ MPa.

The bending strength of similar slabs has been found to have a minimum value $m_R = 4.0$ MPa with a coefficient of variation, cov = 20% . It is desired to evaluate the probability of failure, P_f.

The slab self-weight is not relevant for the effect under study so it can be neglected.

In this way the variance for the resistance is given by: $s_R^2 = (0.20 \times 4.0)^2 = 0.64$ MPa.

Hence: $m_Z = 4 - 3 = 1; s_z^2 = 0.64 + 1.0 = 1.64$.

Therefore $\beta = \frac{m_z}{s_z} = \frac{1}{1.64} \approx 1.2806(3)$ and according (9) $P_f = \Phi(-1.28063) \approx 10.0\%$.

The above result is valid only under the limiting assumptions of the normal distribution of both fundamental variables R and S. In a more general case, when the variables have a general 'non normal' distribution, the probability of failure P_f cannot be determined using Equation (10), but only as first estimates. In this case, however, the probability of failure may still be determined following Equation (5) and or any specialized software.

6. Safety Factors and Characteristic Values

The traditional deterministic measures of limit state "violation", namely the factor of safety and the load factor, can be related directly to the probability P_f of limit state violation.

Analytically this is established more easily for the basic "one-resistance one-load-effect case" when R and S are both normally distributed.

For resistances, R_k the so called characteristic values, are defined on the low side of the mean resistance;

$$R_k = m_R \times (1 - k_R \times V_R) \qquad (12)$$

where R_k is the characteristic resistance, m_R the mean resistance, V_R the coefficient of variation for R and k_R a constant. This is based on the normal distribution on which R_k is the value of resistance below which only say 5% of slab samples will fail. Also, for the standardized normal distribution function it follows that:

$$0.05 = \Phi\left(\frac{R_k - m_R}{s_R}\right) \qquad (13)$$

for a 5% "one-sided tail", $k_{0.05} = \frac{(m_R - R_k)}{s_R} \approx 1.645$.

EC 1990 defines 3 classes for the importance or consequence for loss of human life, or economic, social consequences. Class CC1 – less grievous - is taken for the basis for the calculation of safety factors as proposed by Camposinhos [5] where the corresponding probability of failure is $1{,}33\times10^{-5}$, *i.e.* the reliability index equals to 3,21. These values are set for negligible or small environmental consequences as stated in EC 1990 [6] and for a 50 years period the probability of failure is roughly 0,5/1000 which value is very close of that proposed by Malcolm J. Faddy et al [7]. The partial factors for dimension stone cladding depend on the coefficient of variation (*cv*) of the flexural strength and a *cv* = 10% for slabs dimensions. For its determination a Weibull distribution type is assumed for the lateral actions (wind pressure and seismic forces) with *cv*=40%. Safety partial factor were calibrated with CodeCal 03.01 software issued by JCSS – Joint Committee on Structural Safety. Results are summarized in Table 1.

Table 1 – Partial Safety Factors for dimension stone cladding design

Coefficient of Variation(c*v*)	Partial Safety Factor (γ_M)
$v < 10\%$	1,5
$10\% \le v < 20\%$	2,4
$10\% \le v < 20\%$	2,9
$25\% \le v < 30\%$	3,8
$30\% \le v < 35\%$	4,8
$35\% \le v < 40\%$	6,1

References

[1] Marble Institute of America. Dimension Stone Design Manual, VII. Ohio, Cleveland: Marble Institute of America, 2007. Item: 11050 Marble Institute of America.

[2] Indiana Limestone Institute. Indiana Limestone Handbook. 21 st. Bedford: Indiana Limestone Handbook, Inc., 1998.

[3] National Building Granite Quarries Association Inc. Standard Specifications for Architectural Granite. Washington: National Building Granite Quarries Association Inc., 2007.

[4] **Beall, Christine**. Selecting a safety factor for Stone. Boston: Aberdeen Group Publication # M900176, 1990.

[5] **Camposinhos, Rui de Sousa**. Revestimentos em Pedra Natural - Dimensionamento e Projecto. Lisboa: Edições Sílabo, 2009.ISBN: 978-972-618-561-1.

[6] EN 1990 Eurocode 0: Basis of structural design. Brussels : European Committee for Standardization, 2001. ICS 91.010.30.

[7] **Malcolm J. Faddy**, Richard J. Wilson, and Gerry M. Winter, The Determination of the Design Strength of Granite Used as External Cladding for Buildings, Case Studies in Reliability and Maintenance, Edited by W. R. Blischke and D. N. P. Murthy. 111,ISBN 0-471-41373-9 © 2003 John Wiley and Sons, Inc